

$$\frac{\text{SUBST-L}}{a \doteq T_1 \quad a < T_2} \quad \frac{\text{SUBST-R}}{a \doteq T_1 \quad T_2 < a} \quad \frac{\text{SUBST-EQUAL}}{a \doteq T_1 \quad a \doteq T_2} \quad \frac{\text{SWAP}}{T_1 \doteq T_2}$$

$$\frac{\text{UNFOLD}}{b \doteq C\langle T_1 \dots T_n \rangle} \quad \frac{\text{UNFOLD}'}{b < C\langle T_1 \dots T_n \rangle} \quad \frac{\text{SUBST-PARAM}}{T' \doteq S \quad T \doteq C\langle T_1 \dots, T', \dots T_n \rangle}$$

$$\frac{\text{SUBST-PARAM}'}{T' \doteq S \quad b < C\langle T_1 \dots, T', \dots T_n \rangle} \quad \frac{\text{S-OBJECT}}{a < \text{Object}}$$

$$\frac{\text{MATCH}}{a < N_1 \quad a < N_2 \quad N_1 << N_2} \quad \frac{\text{ADOPT}}{a < b \quad b < T}$$

$$\frac{\text{ADAPT}}{N_1 < C\langle T_1 \dots T_n \rangle \quad N_1 <: C\langle S_1 \dots S_n \rangle} \quad \frac{\text{REDUCE}}{C\langle S_1 \dots S_n \rangle \doteq C\langle T_1 \dots T_n \rangle}$$

$$\frac{\text{MATCH}}{a < N_1 \quad a < N_2 \quad N_1 << N_2} \quad \frac{\text{ADOPT}}{a < b \quad b < T}$$

$$\frac{\text{ADAPT}}{N_1 < C\langle T_1 \dots T_n \rangle \quad N_1 <: C\langle S_1 \dots S_n \rangle} \quad \frac{\text{REDUCE}}{C\langle S_1 \dots S_n \rangle \doteq C\langle T_1 \dots T_n \rangle}$$

$$\frac{\text{SUPER}}{N < a \quad N <: N'}$$

$$\frac{\text{SPLIT-L}}{a < b \quad a < N} \quad \frac{\text{SPLIT-R}}{a < b \quad a < N}$$

$$\frac{\text{SOLUTION}}{a \doteq G} \quad \frac{\text{SOLUTION-GEN}}{a < C_1, \dots, a < C_n \quad \forall i: C_m << C_i}$$

$$\frac{\text{FAIL}}{C\langle \dots \rangle \doteq D\langle \dots \rangle \quad C \neq D} \quad \frac{\text{FAIL-GENERIC}}{X \doteq T \quad X \neq T} \quad \frac{\text{FAIL-SIGMA}}{a \doteq N \quad a \in N}$$

$$\frac{\text{FAIL}}{C\langle \dots \rangle \doteq D\langle \dots \rangle \quad C \neq D} \quad \frac{\text{FAIL-GENERIC}}{X \doteq T \quad X \neq T} \quad \frac{\text{FAIL-SIGMA}}{a \doteq N \quad a \in N}$$

$$\frac{\text{FAIL}}{C\langle \dots \rangle \doteq D\langle \dots \rangle \quad C \neq D} \quad \frac{\text{FAIL-GENERIC}}{X \doteq T \quad X \neq T} \quad \frac{\text{FAIL-SIGMA}}{a \doteq N \quad a \in N}$$

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$$\frac{\text{FAIL}}{C\langle \dots \rangle \doteq D\langle \dots \rangle \quad C \neq D} \quad \frac{\text{FAIL-GENERIC}}{X \doteq T \quad X \neq T} \quad \frac{\text{FAIL-SIGMA}}{a \doteq N \quad a \in N}$$

$c ::= T < T \mid T \doteq T$ Constraint
 $T ::= a \mid N$ Type placeholder or Type
 $N ::= C\langle \bar{T} \rangle$ Class Type containing type placeholders
 $G ::= C\langle \bar{G} \rangle$ Class Type not containing type placeholders

$$\frac{\text{S-REFL}}{T <: T} \quad \frac{\text{S-TRANS}}{T_1 <: T_2 \quad T_2 <: T_3} \quad \frac{\text{S-VAR}}{A <: \Delta(A)} \quad \frac{\text{S-CLASS}}{\text{class } C\langle \bar{X} \rangle \triangleleft N}$$

$$\frac{\text{N-REFL}}{C << C} \quad \frac{\text{N-TRANS}}{C_1 << C_2 \quad C_2 << C_3} \quad \frac{\text{N-CLASS}}{\text{class } C\langle \dots \rangle \triangleleft D\langle \dots \rangle}$$

$$\frac{\text{ADAPT}}{N_1 < C\langle T_1 \dots T_n \rangle \quad N_1 <: C\langle S_1 \dots S_n \rangle} \quad \frac{\text{REDUCE}}{C\langle S_1 \dots S_n \rangle \doteq C\langle T_1 \dots T_n \rangle}$$

$$\frac{\text{SUPER}}{N < a \quad N <: N'}$$

$$\frac{\text{SPLIT-L}}{a < b \quad a < N} \quad \frac{\text{SPLIT-R}}{a < b \quad a < N}$$

$$\frac{\text{SOLUTION}}{a \doteq G} \quad \frac{\text{SOLUTION-GEN}}{a < C_1, \dots, a < C_n \quad \forall i: C_m << C_i}$$

$$\frac{\text{FAIL}}{C\langle \dots \rangle \doteq D\langle \dots \rangle \quad C \neq D} \quad \frac{\text{FAIL-GENERIC}}{X \doteq T \quad X \neq T} \quad \frac{\text{FAIL-SIGMA}}{a \doteq N \quad a \in N}$$

$$\frac{\text{FAIL}}{C\langle \dots \rangle \doteq D\langle \dots \rangle \quad C \neq D} \quad \frac{\text{FAIL-GENERIC}}{X \doteq T \quad X \neq T} \quad \frac{\text{FAIL-SIGMA}}{a \doteq N \quad a \in N}$$

$$\frac{\text{FAIL}}{C\langle \dots \rangle \doteq D\langle \dots \rangle \quad C \neq D} \quad \frac{\text{FAIL-GENERIC}}{X \doteq T \quad X \neq T} \quad \frac{\text{FAIL-SIGMA}}{a \doteq N \quad a \in N}$$

$$\frac{\text{FAIL}}{C\langle \dots \rangle \doteq D\langle \dots \rangle \quad C \neq D} \quad \frac{\text{FAIL-GENERIC}}{X \doteq T \quad X \neq T} \quad \frac{\text{FAIL-SIGMA}}{a \doteq N \quad a \in N}$$

$$\frac{\text{FAIL}}{C\langle \dots \rangle \doteq D\langle \dots \rangle \quad C \neq D} \quad \frac{\text{FAIL-GENERIC}}{X \doteq T \quad X \neq T} \quad \frac{\text{FAIL-SIGMA}}{a \doteq N \quad a \in N}$$

$$\frac{\text{FAIL}}{C\langle \dots \rangle \doteq D\langle \dots \rangle \quad C \neq D} \quad \frac{\text{FAIL-GENERIC}}{X \doteq T \quad X \neq T} \quad \frac{\text{FAIL-SIGMA}}{a \doteq N \quad a \in N}$$

Result:

Fail:

$$\frac{\text{FAIL}}{a < N_1 \quad a < N_2 \quad \text{not } N_1 << N_2 \quad \text{not } N_2 << N_1}$$